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Relation Between the Rate of Growth of Loudness and the Intensity DL

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Hearing loss of cochlear origin is usually associated with an abnormal growth of loudness known as *loudness recruitment*. It is commonly assumed that when loudness increases more rapidly than normal, a smaller than normal change in intensity will be required for a just-noticeable difference (JND). According to this reasoning, the difference limen (DL) for intensity might be expected to be much smaller in ears with loudness recruitment. However, the fact that the observed intensity DL is not much smaller in ears with loudness recruitment has led some to conclude that there is no relation between the rate of growth of loudness and the intensity DL. This apparent contradiction can be eliminated by correct interpretation of the associated variables. According to the theory of Hellman and Hellman (1990), the intensity JND for a tone is inversely proportional to the slope of the loudness with respect to intensity; however, it is the slope of *log-loudness* with respect to *log-intensity* that grows more rapidly in recruiting ears, not the slope of *loudness* with respect to *intensity*. An example is presented in which a simulated 40 dB hearing loss exhibits a significant amount of loudness recruitment, but only a slight decrease in the intensity JND.

INTRODUCTION

The intensity of a sound is a physical variable that is defined as the square of the pressure divided by the acoustic impedance. The just-noticeable difference (JND) in sound intensity ΔI is determined by finding the smallest intensity $I + \Delta I$ that a subject can discriminate from the intensity I, 50% of the time.

The loudness of a sound $L(I)$ is a perceived quantity that is closely linked to intensity. Any detectable increment in the intensity of a sound has a corresponding increment in loudness. The loudness JND $\Delta L(I)$ is defined as the change in loudness $L(I)$ corresponding to $\Delta I(I)$.

The relation between the intensity JND ΔI and the loudness JND ΔL is a classical problem in auditory theory. This relation has been studied extensively, but remains somewhat controversial. Two prevailing theories are the "proportional JND" theory and the "equal loudness, equal JND" theory.

The "proportional JND" theory, first described by Riesz (1933), postulates that two tones will be judged equally loud when their intensities correspond to equal JND counting ratios. The JND counting ratio is defined as the number of JNDs between threshold intensity and the test intensity divided by the number of JNDs between threshold intensity and some reference intensity, known to correspond to equal loudness. Riesz supported this theory by using a single equal loudness contour from Fletcher and Munson (1933) to reconstruct an entire family of equal loudness contours.

The "equal loudness, equal JND" theory was described by Zwislocki and Jordan (1986) as an alternative to the "proportional JND" theory. This theory is based on the empirical observation that size of the JND appears to be unrelated to the "slope of loudness." However, because the "slope of loudness" was evaluated in terms of dB/dB, it is not clear that the loudness JND should depend on this quantity. Although some measurements appear to support the "equal loudness, equal JND" theory (e.g., Stillman, Zwislocki, Zhang, & Cafaratti, 1993), other measurements do not support this theory (e.g., Rankovic, Viemeister, Fantini, Cheeseman, & Uchiyama, 1988).

In this chapter, we use the term *log-loudness slope* to refer to the growth of loudness expressed in terms of dB/dB and distinguish this quantity from the *loudness slope* (defined as the derivative of the loudness function with respect to intensity). We demonstrate, for the case of a 1 kHz tone with quiet background, that an increase in the log-loudness slope does not necessarily imply an increase in loudness slope.

DERIVATION OF THE RELATION BETWEEN LOUDNESS AND THE INTENSITY JND

A derivation of the relation between loudness and the intensity JND is summarized in this section in order to establish its validity. For a more complete description of this relation, see Allen and Neely (1996).

The Taylor series expansion for $L(I)$ is

$$L(I+\Delta I) = L(I) + \Delta I \left.\frac{dL}{dI}\right|_I + \text{HOT}$$

When the intensity JND ΔI is small, we can ignore the higher-order terms (HOT) and approximate the loudness JND by

$$\Delta L = \Delta I \left.\frac{dL}{dI}\right|_I \tag{1}$$

where, by definition, $\Delta L \equiv L(I + \Delta I) - L(I)$. This shows that the loudness JND ΔL is related to the intensity JND ΔI by the slope of the loudness function, evaluated at I.

We assume that the *single-trial loudness* is a random variable that is proportional to the total number of neural spikes that result from the presentation of a tone, and that the neural spikes are generated by a Poisson process. The ratio of the variance of the loudness σ_L^2 to the average loudness $L(I)$ is called the *Fano factor*,

$$r = \sigma_L^2 / L \tag{2}$$

We assume that $r = 1$ for levels below the refractory limit, because this is one of the special properties of a Poisson process.

From signal detection theory, we expect that

$$\Delta L = d'\sigma_L \tag{3}$$

where d' is a constant that depends on the design of the signal detection task. Combining equations 1, 2, and 3 gives us the desired relation between loudness and the intensity JND

$$\Delta I(I) = \frac{h\sqrt{L(I)}}{\frac{dL}{dI}} \tag{4}$$

where $h \equiv d'\sqrt{r}$. We can also express this relation in terms of the Weber fraction $J \equiv \Delta I/I$

$$J(I) = \frac{h\sqrt{L(I)}}{I\frac{dL}{dI}} \tag{5}$$

Because equation 5 was first derived by Hellman and Hellman (1990), we refer to it as the Hellman and Hellman Loudness Limen (HHLL) equation. On the lefthand side of equation 5, we have the Weber fraction or relative intensity JND. On the righthand side, we have terms that can be computed from the loudness-intensity function. Because loudness and the intensity JND are measurable quantities, we can test the validity of the HHLL equation using data from the literature.

In Fig. 13.1, we use the HHLL equation to predict the Weber fraction from the Fletcher-Munson (1933) loudness-intensity function, and compare this prediction with the direct measurements of Riesz (1928) for a 1 kHz tone. The loudness function $L(I)$ in the upper-left panel is from Table III of Fletcher and Munson (1933). The slope of the loudness in the upper-right panel was computed numerically from $L(I)$. The solid line in the lower-left panel was computed using equation 5 with $h = 2.4$. The excellent agreement between the predicted curve and the measured data provides strong support for the validity of the HHLL equation. The measurements of Jesteadt, Wier, and Green (1977) are also included in Fig. 12.1 for comparison. The agreement between the predicted Weber fraction and the Jesteadt et al. measurements is good, but could be improved by setting $h = 3$ (Allen & Neely, 1996).

The log-loudness slope (lower-right panel) indicates the slope of the loudness function when it is plotted on log-log axes as it is in the upper-left panel of this figure. Note that the Weber fraction predicted by equation 5 depends on the loudness slope, but does not depend on the log-loudness slope. We see in the next section that an increase in the log-loudness slope (due to loudness recruitment) does not necessarily imply an increase in the loudness slope.

HEARING LOSS SIMULATION

Here we simulate a hearing loss of cochlear origin as a level dependent attenuation of the sound intensity delivered to the auditory nerve. Accordingly, we define an impaired loudness function $L'(I)$ for I above the impaired threshold I_1.

$$L'(I) = L(I \cdot A),\ I > I_1 \tag{6}$$

where $A(I)$ is a level-dependent attenuation. For illustration, we can simulate a hearing loss of $-10\log_{10}(A_0)$ decibels by selecting the attenuation function to be

$$A(I) = A_0^{\,(A_0 I/I_0)^{\tau}},\ I > I_1 \tag{7}$$

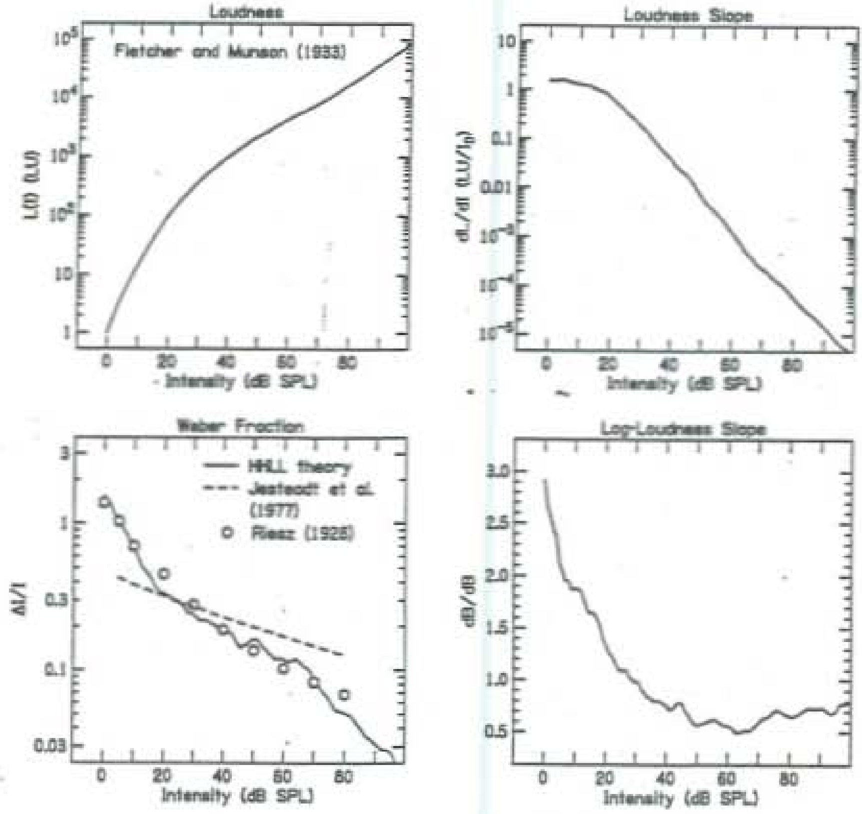

FIG. 13.1. Comparison between predicted and measured intensity JNDs. The upper-left panel shows the loudness function for a 1 kHz tone measured by Fletcher and Munson (1933). The upper-right panel shows the slope of the loudness function, defined as the derivative of loudness with respect to intensity. Threshold intensity for the normal ear is taken to be 0 dB SPL. The lower-left panel compares the predicted intensity JND, obtained from the loudness function and the HHLL equation (with $h = 2.4$), with the measured values of Riesz (1928) and Jesteadt, Wier, and Green (1977). The lower-right panel shows the log-loudness slope, defined as the increase in the function 20 times the log (base 10) of the loudness due to a 1 dB increase in intensity.

where I_0 is the threshold intensity for normal hearing and the exponent p controls the rate of growth of the log-loudness function. For example, we can simulate a 40 dB hearing loss by selecting $A_0 = 10^{-4}$ and obtain a somewhat realistic growth of loudness with $p = 0.217$. This example is illustrated in Fig. 13.2 in the form of a loudness balance graph where the difference between the "better ear" and "poorer ear" is defined by the attenuation function $A(I)$.

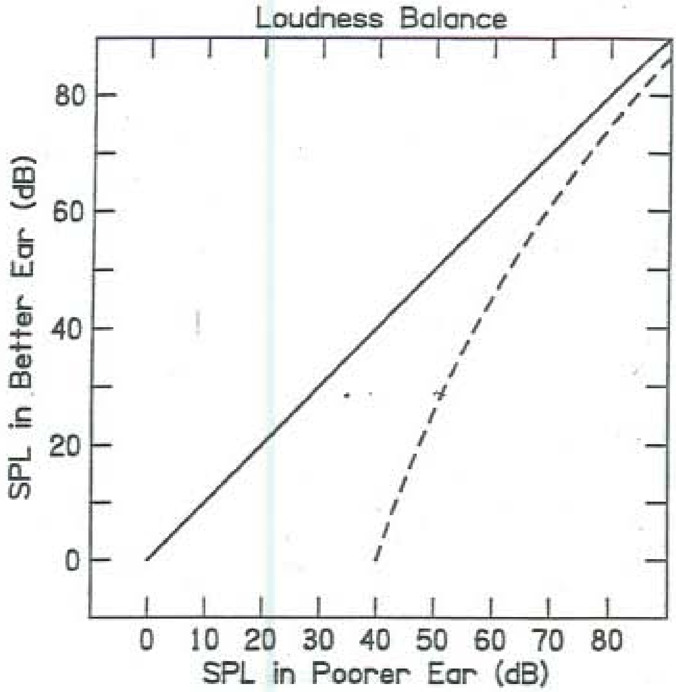

FIG. 13.2. Loudness balance for simulated hearing loss. The difference between the "better-ear" and "poorer ear" is defined by the attenuation function $A(I)$ in equation 7, with $A_0 = 10^{-4}$, $p = 0.217$, and $I_1 = 10^4 I_0$.

In each panel of Fig. 13.3, we compare the impaired loudness function $L'(I)$ (dashed line) with the normal loudness function $L(I)$ (solid line). Note that the loudness slope in the upper-right panel for the impaired ear is less than (or equal to) the normal ear over the entire range of intensity, while the log-loudness slope in the lower-right panel is greater for the impaired ear than for the normal ear. Clearly, the fact that a loudness function has a greater log-loudness slope does not imply that it also has a greater (linear) loudness slope.

The predicted Weber fraction for the impaired ear in the lower-left panel of Fig. 13.3 is larger near the impaired threshold of 40 dB SPL, slightly lower in the range from 50 to 70 dB, and nearly the same above 70 dB. This result is similar to measurements of intensity JNDs in impaired ears (e.g., Stillman et al., 1993). The predicted Weber fraction curves are replotted in Fig. 13.4 as a function of sensation level and as a function of loudness. The agreement between the normal and impaired Weber fraction curves appears to be best when plotted as a function of loudness, in which case the Weber fraction for the impaired ear appears to be consistently lower than that for the normal ear.

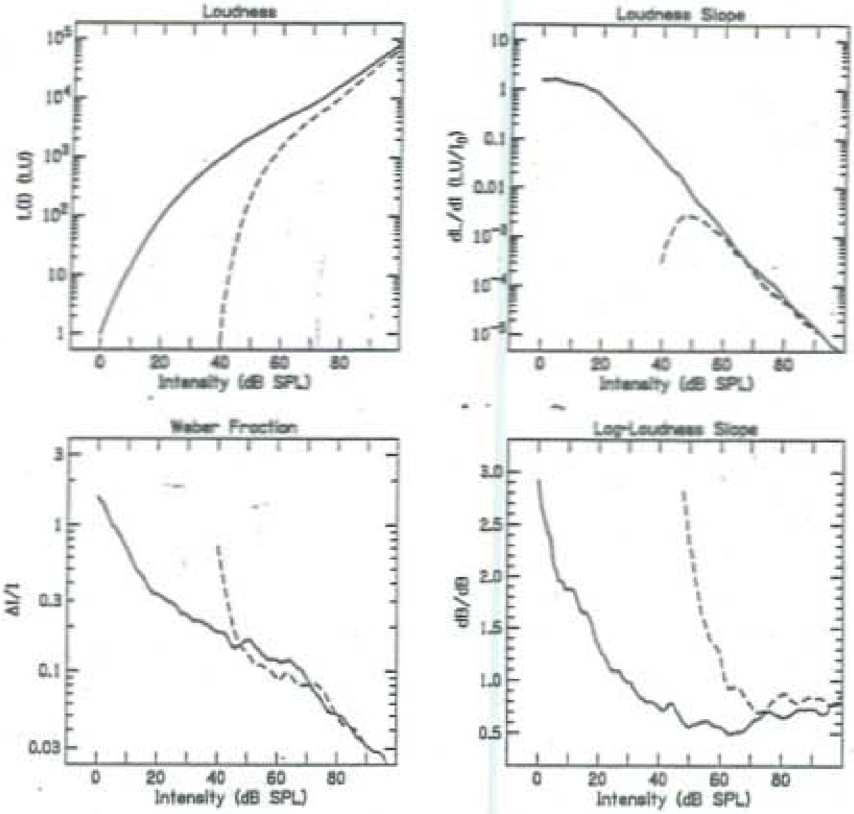

FIG. 13.3. Comparison between normal and impaired loudness functions. The solid lines represent the loudness function for a normal ear, based on the measurements of Fletcher and Munson (1933). The dashed lines represent the loudness function for an impaired ear defined by equation 6.

DISCUSSION

The comparison in Fig. 13.1 between the classic loudness function of Fletcher and Munson (1933) and the classic intensity JND data of Riesz (1928) supports the validity of the HHLL equation. This comparison is discussed in more detail in Allen and Neely (1996). The HHLL equation assumes that the variance in the single-trial loudness is due to the spike generation process (internal noise) and does not include contributions to the variance from thermal noise within the cochlea or the presence of noise maskers (external noise). An investigation of the effect of masking on the intensity JND would require extending the HHLL theory.

The simulation of a 40 dB hearing loss as a level-dependent attenuation in equation 6 is consistent with impairment of cochlear outer hair cells,

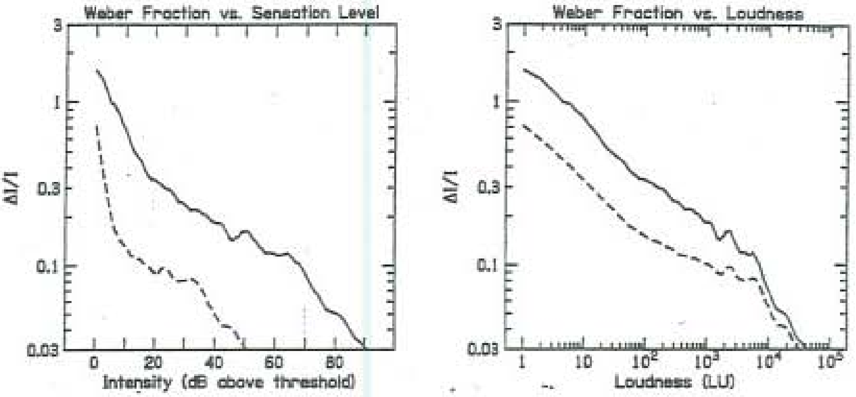

FIG. 13.4. Weber fraction versus sensation level and loudness for normal and impaired ears. The solid lines represent a normal ear and the dashed lines represent an impaired ear. The Weber fraction is plotted as a function of sensation level in the left panel and as a function of loudness in right panel. These are the same curves that were plotted as a function of SPL in the lower-left panel of Fig. 13.3.

which normally act in a level-dependent manner to effectively compress the dynamic range of vibrations delivered to inner hair cells by about 40 dB (Allen, 1995). This simulation is also consistent with observed loudness balance functions (e.g., Stillman et al., 1993), which show the largest differences between normal and impaired ears in the intensity required to match the loudness of tones near threshold, and much smaller differences well above threshold.

The agreement between the normal and impaired intensity JND functions was best for the simulated hearing loss when plotted as a function of loudness. This observation is in general agreement with Zwislocki's hypothesis (Zwislocki & Jordan, 1986) that the intensity JND is equal when the loudness is equal. However, the fact that the simulated intensity JND for the impaired ear is always lower than the normal ear is inconsistent with measurements in impaired ears that show the intensity JND to be sometimes greater than in the normal ear at the same loudness (e.g., Stillman et al., 1993). This may represent a limitation of the HHLL theory.

In our simulation, the slope of the loudness function was smaller for the impaired ear than for the normal ear at all levels when the comparison was made at equal intensity; however, when the comparison was made at equal loudness, the slope of the loudness for the impaired ear was always larger. This will be the case for any choice of attenuation function, provided that it is largest near threshold and decreases at high levels. Thus, it seems that the HHLL equation will always predict a smaller intensity JND for an impaired ear at equal loudness, although the difference between impaired and normal may be negligibly small in many instances.

The fact that the HHLL equation predicts a slightly smaller JND for the impaired ear is similar to the trend observed in some measurements. Stillman et al. (1993) observed that their results "suggest that intensity jnd's in the poorer ear with pathological loudness recruitment may tend to be lower at medium and high sensation levels than in the contralateral, better ear" (p. 433). Likewise, Schroder, Viemeister, and Nelson (1994) observed that "the Weber fraction was sometimes smaller in the cochlear-impaired than in normal-hearing listeners" (pp. 2, 683).

The loudness balance function for impaired ears is often steeper than the example shown in Fig. 13.2. The steepness of the loudness balance function in our simulated hearing loss is controlled by the parameter p. Increasing the value of p will increase the loudness slope and cause the predicted Weber fraction to decrease. For example, if the value of p is increased by a factor of two, the Weber fraction, when plotted as a function of loudness, will decrease by about 20%.

The major point of this chapter is that the larger log-loudness slope expected for an impaired ear does not imply that the loudness slope is also larger. This point is clearly illustrated by the comparison between the normal and impaired ears in Fig. 13.3, where the loudness slope was smaller (or equal) at all intensities even though the log-loudness slope was larger at all intensities. Thus, it is misleading (and incorrect) to describe loudness recruitment as a more rapid growth in loudness. It would be better to describe loudness recruitment as a loss of dynamic range compression. This is more than just a matter of semantics. The association of loudness recruitment with rapid loudness growth has brought with it an expectation of much lower intensity JNDs for the impaired ear, and led some to conclude that an association between loudness slope and the intensity JND was inconsistent with experimental observations. A disassociation between loudness recruitment and rapid growth of loudness may help to avoid this confusion and promote acceptance of the HHLL equation.

CONCLUSION

Loudness recruitment should not be defined as "a more rapid growth in loudness." Loudness recruitment represents a loss of the dynamic-range compression normally provided by outer hair cells. This results in a steeper log-loudness slope; however, it does not imply that the impaired ear has a steeper loudness slope when the comparison is made at the same intensity.

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